

# Electron capture processes in strongly coupled semiclassical plasmas

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**Abstract.** The electron captures by projectile ions from hydrogenic ions are investigated in strongly coupled semiclassical plasmas. The electron capture radius by the projectile ion is obtained by the effective screened pseudopotential model taking into account both the plasma screening and quantum effects. The semiclassical version of the Bohr-Lindhard method is applied to obtain the electron capture probability. The impact-parameter trajectory analysis is applied to the motion of the projectile ion in order to visualize the electron capture radius and capture probability as functions of the impact parameter, thermal de Broglie wavelength and Debye length. The results show that the quantum and plasma screening effects significantly reduce the electron capture probability and the capture radius. It is found that the electron capture position is shifted to the core of the projectile ion with increasing the thermal de Broglie wavelength. It is also found that the quantum effects on the electron capture probability are more significant than the collective screening effects on the electron capture probability. The electron capture probability is found to be significantly increased with an increase of the charge.

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## 1 Introduction

The electron capture process [1–11] due to charged particle collisions has been of great interest since this process is one of the most fundamental processes in atomic physics. This electron capture process has been investigated widely using various methods [1,9] depending on the physical properties of the collision system. It has been known that the Bohr-Lindhard method [2,6] is quite reliable for evaluating the electron capture cross-section when the relative collision velocity  $v_P$  of the projectile ion is greater than the ground orbital velocity  $v_{Z_T} (= Z_T e^2 / \hbar)$  of the hydrogenic target ion with nuclear charge  $Z_T$ . Recently, the atomic collision and radiation processes in plasmas have been of great interest since these processes can be used for plasma diagnostics [7,10]. It has been known that the Debye-Hückel screened potential describes the physical properties of a low density plasma and corresponds to a pair correlation approximation [12]. Recently, the importance of study of various physical properties of strongly coupled plasmas such as the inertial confinement fusion plasmas and the interiors of the astrophysical compact objects has considerably increased. It is quite evident that the physical properties of matter existing un-

der such strongly coupled plasmas differ radically from the properties of weakly coupled ideal plasmas since the Coulomb coupling parameter is greater than unity in these strongly coupled plasmas. In addition, the interaction potential in strongly coupled semiclassical plasmas cannot be described by the Debye-Hückel type potential because of strong nonideal particle interactions [13–20] due to the collective plasma screening and quantum effects. This collective effect is mainly caused by the many-body strong correlation interactions in dense plasmas. Then, the electron capture processes in strongly coupled plasmas would be quite different from those in weakly coupled ideal plasmas. Thus, in this paper we investigate electron capture processes by projectile ions from hydrogenic target ions in strongly coupled semiclassical plasmas. The analytic pseudopotential model [20] including the quantum effects and the collective plasma screening effects is applied to describe the interaction between the projectile ion and the released electron in strongly coupled semiclassical plasmas. The semiclassical version of the Bohr-Lindhard model [8] is used to obtain the electron capture radius and the electron capture probability including the collective plasma screening and quantum effects. The impact-parameter trajectory analysis is applied to the path of the projectile ion in order to visualize the quantum and collective plasma screening effects on the electron capture radius and probability as functions of the impact parameter, thermal de Broglie wavelength and Debye length.

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In Section 2, we discuss the semiclassical expression of the electron capture cross-section. We obtain the analytic expression of the electron capture radius by the projectile ion from the hydrogenic target ion in strongly coupled plasmas using the screened pseudopotential model. In Section 3, we derive the closed form of the scaled electron capture probability as a function of the impact parameter, thermal de Broglie wavelength, Debye length, and collision energy in accordance with the semiclassical version of the Bohr-Lindhard method and the effective interaction potential. We also investigate the variation of the scaled electron capture probability with changing the physical parameters of strongly coupled semiclassical plasmas. Finally, in Section 4, summary and discussions are given.

## 2 Capture radius

The cross-section  $\sigma_C$  for the electron capture process can be obtained by the Bohr-Lindhard method and the impact-parameter trajectory analysis [4]:

$$\sigma_C = \int d^2\mathbf{b} P_C(b), \quad (1)$$

where  $\mathbf{b}$  is the impact parameter and  $P_C(b)$  is the electron capture probability. In the Bohr-Lindhard method [6], it has been known that the electron capture happens when the distance between the projectile ion and the released electron is smaller than the electron capture radius  $R_C$ . This electron capture radius is usually determined by equating the kinetic energy of the released electron in the frame of the projectile ion and the binding energy provided by the projectile ion.

The pseudopotential models of the particle interaction in strongly coupled plasmas have been proposed by several authors [13, 14, 16, 17, 20]. Very recently, the analytic expression of the effective screened potential [20] of the charged particle interaction in strongly coupled semiclassical plasmas taking into account both the plasma screening effects and the quantum effects of diffraction has been obtained on the basis of the dielectric response function analysis. Based on the screened pseudopotential model [20], the interaction potential  $V(r)$  between the projectile ion with nuclear charge  $Z_P$  and the released electron in strongly coupled semiclassical plasmas for the region of  $2\lambda < \Lambda$  would be represented as

$$V(r, \lambda, \Lambda) = -\frac{Z_P e^2}{\sqrt{1 - 4\lambda^2/\Lambda^2}} \left[ \frac{e^{-A(\lambda, \Lambda)r}}{r} - \frac{e^{-B(\lambda, \Lambda)r}}{r} \right], \quad (2)$$

where  $\lambda (= \hbar/\sqrt{2\pi m k_B T})$  is the thermal de Broglie wavelength,  $k_B$  stands for the Boltzmann constant,  $T$  is the plasma temperature,  $\Lambda$  is the Debye length, and the parameters  $A(\lambda, \Lambda)$  and  $B(\lambda, \Lambda)$  are, respectively, defined as  $A^2(\lambda, \Lambda) = (1 - \sqrt{1 - 4\lambda^2/\Lambda^2})/(2\lambda^2)$  and  $B^2(\lambda, \Lambda) = (1 + \sqrt{1 - 4\lambda^2/\Lambda^2})/(2\lambda^2)$ . This effective interaction potential has been known to be reliable for strongly coupled plasmas when the plasma density  $n \approx 10^{20}$ – $10^{24}$  cm<sup>-3</sup>

and the temperature  $T \approx 10^3$ – $10^7$  K, i.e., the plasma coupled parameter  $\Gamma > 1$  [20]. Then, the electron capture radius  $R_C$  in strongly coupled semiclassical plasmas can be determined by the following relation:

$$\frac{Z_P e^2}{\sqrt{1 - 4\lambda^2/\Lambda^2}} \left[ \frac{e^{-A(\lambda, \Lambda)R_C}}{R_C} - \frac{e^{-B(\lambda, \Lambda)R_C}}{R_C} \right] \cong \frac{1}{2} m v_P^2, \quad (3)$$

since the kinetic energy of the released electron in the frame of the projectile ion has to be smaller than the binding energy provided by the projectile proton, where  $m$  is the electron mass and  $v_P$  is the relative collision velocity. Here, the perturbation analysis can be applied to obtain the electron capture radius since the capture radius is usually smaller than the Debye length  $\Lambda$  and  $A(\lambda, \Lambda) \ll B(\lambda, \Lambda)$  for the domain  $2\lambda < \Lambda$  in order to have real values of the parameters  $A(\lambda, \Lambda)$  and  $B(\lambda, \Lambda)$ . After some algebra, the analytic form of the scaled electron capture radius ( $\bar{R}_C$ ), i.e., the capture radius in units of  $a_{Z_P}$ , including the collective plasma screening and quantum effects is then found to be

$$\begin{aligned} \bar{R}_C(\bar{E}, \bar{\lambda}, a_\Lambda) &= R_C(\bar{E}, \bar{\lambda}, a_\Lambda)/a_{Z_P}, \\ &= \ln \left( 4 - \sqrt{21 - 12F(\bar{E}, \bar{\lambda}, a_\Lambda)} \right)^{-1/\bar{B}(\bar{\lambda}, a_\Lambda)}, \end{aligned} \quad (4)$$

where  $a_{Z_P} (= a_0/Z_P)$  is the Bohr radius of the hydrogenic ion with nuclear charge  $Z_P$ ,  $a_0 (= \hbar^2/m_e^2)$  is the Bohr radius of the hydrogen atom,  $\bar{E} (= m v_P^2/2Z_P^2 R_y)$  is the scaled collision energy,  $R_y (= m e^4/2\hbar^2 \approx 13.6$  eV) is the Rydberg constant,  $\bar{\lambda} (= \lambda/a_{Z_P})$  is the scaled de Broglie wavelength,  $a_\Lambda (= a_{Z_P}/\Lambda)$  is the scaled reciprocal Debye length,  $\bar{B}(\bar{\lambda}, a_\Lambda) \equiv B(\bar{\lambda}, a_\Lambda)a_{Z_P}$ , and the function  $F(\bar{E}, \bar{\lambda}, a_\Lambda)$  is

$$F(\bar{E}, \bar{\lambda}, a_\Lambda) = \frac{(\bar{E}/\sqrt{2})\bar{\lambda}\sqrt{1 - (2\bar{\lambda}a_\Lambda)^2} + \sqrt{1 - \sqrt{1 - (2\bar{\lambda}a_\Lambda)^2}}}{\sqrt{1 + \sqrt{1 - (2\bar{\lambda}a_\Lambda)^2}}}. \quad (5)$$

The dependence of the electron capture radius on the collective plasma screening and quantum effects can be investigated by equations (4) and (5) since the capture radius is mainly determined by the kinetic energy and the interaction potential in strongly coupled plasmas, i.e., the classical over the barrier analysis [6].

## 3 Capture probability

Using the semiclassical version of the Bohr-Lindhard method [6], the electron capture probability for the target system with nuclear charge  $Z_T$  is given by

$$P_C = \int d^3\mathbf{r}_T \frac{2t_C}{\tau(r_T)} |\Psi(\mathbf{r}_T)|^2, \quad (6)$$

where the electron release time  $\tau(r_T)$  ( $\cong \hbar/E(r_T) = 2r\hbar/Z_T e^2$ ) is given by the uncertainty principle,  $t_C$  is the electron capture time,  $\Psi(\mathbf{r}_T)$  and is the wave function of the electron in the target ion. It is obvious that the straight-line trajectory analysis is more reliable for heavy ( $M$ ) projectiles rather than light particles ( $m$ ) due to large mass ratios ( $M/m \gg 1$ ) [21,22]. The electron capture time  $t_C$  is then obtained as

$$t_C(\rho) = \begin{cases} 2(R_C^2 - \rho^2)^{1/2}/v_P, & \text{for } \rho \leq R_C, \\ 0, & \text{for } \rho \geq R_C, \end{cases} \quad (7)$$

where  $\rho$  is the distance between the projectile ion and the released electron. Then, the semiclassical electron capture probability from the electron in the  $1s$  ground state of the target ion becomes

$$P_C = \frac{4Z_T e^2}{\hbar v_P} \int_{\rho \leq R_C} d^3 \mathbf{r}_T \frac{R_C}{r_T} |\Psi_{1s}(\mathbf{r}_T)|^2 (1 - \rho^2/R_C^2)^{1/2}. \quad (8)$$

Here, the factor  $(1 - \rho^2/R_C^2)^{1/2}$  can be approximated by its average value over the integral region since it is a slowly varying function:

$$(1 - \rho^2/R_C^2)^{1/2} = \frac{1}{R_C^2} \int_0^{R_C} d\rho \rho (1 - \rho^2/R_C^2)^{1/2} = \frac{1}{3}. \quad (9)$$

Using the  $1s$  ground state wave function  $\Psi_{1s}(\mathbf{r}_T)$  [23], the semiclassical electron capture probability by the projectile ion from the hydrogenic target ion in strongly coupled plasmas is found to be

$$P_C(b) = \frac{1}{6\pi} \frac{Z_T e^2 R_C}{\hbar v_P a_{Z_T}} G(b, R_C), \quad (10)$$

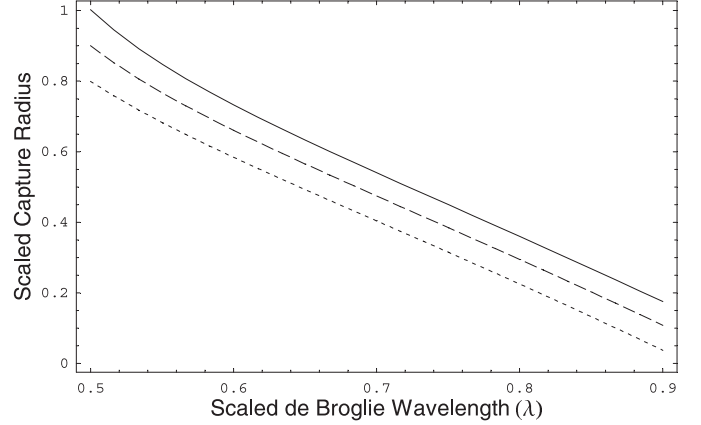
where the integral function  $G(b, R_C)$  is represented in the following form using the cylindrical coordinates  $(\rho, \phi, z)$ :

$$\begin{aligned} G(b, R_C) &= \int_{\rho \leq R_C} d^3 \mathbf{r}_T \frac{\exp(-2r_T/a_{Z_T})}{r_T}, \\ &= \int_0^{R_C} d\rho \rho \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz \frac{\exp(-2r_T(\rho, \phi, z)/a_{Z_T})}{r_T(\rho, \phi, z)}. \end{aligned} \quad (11)$$

This integral function  $G(b, R_C)$  can be evaluated using the inverse Fourier transformation in the momentum space  $\mathbf{q}$  with  $\mathbf{r}_T = \boldsymbol{\rho} + \mathbf{z} + \mathbf{b}$ :

$$\begin{aligned} G(b, R_C) &= \frac{1}{2\pi^2} \int_0^{R_C} d\rho \rho \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz \\ &\quad \times \int d^3 \mathbf{q} \frac{\exp(-i\mathbf{q} \cdot (\boldsymbol{\rho} + \mathbf{z} + \mathbf{b}))}{q^2 + 4/a_{Z_T}^2}, \\ &= 4\pi R_C \int_0^{\infty} dq_{\perp} \frac{J_0(q_{\perp} b) J_1(q_{\perp} R_C)}{q_{\perp}^2 + 4/a_{Z_T}^2}, \end{aligned} \quad (12)$$

where  $q_{\perp}$  is the perpendicular component of the momentum transfer and  $J_n$  the  $n$ th-order Bessel function [24].



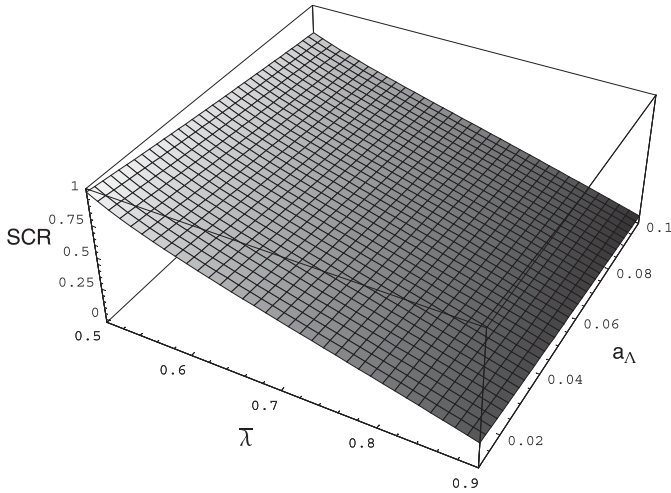
**Fig. 1.** The scaled electron capture radius ( $\bar{R}_C$ ) as a function of the scaled de Broglie wavelength ( $\bar{\lambda}$ ) for various values of the scaled reciprocal Debye length ( $a_A$ ). The solid line represents the electron capture radius for  $a_A = 0.01$ . The dashed line represents the electron capture radius for  $a_A = 0.05$ . The dotted line represents the electron capture radius for  $a_A = 0.1$ .

For the sake of simplicity, we assume that  $Z_P = Z_T \equiv Z$ . Then, the scaled semiclassical electron capture probability by the projectile ion from the ground state of the hydrogenic ion in strongly coupled plasmas is found to be

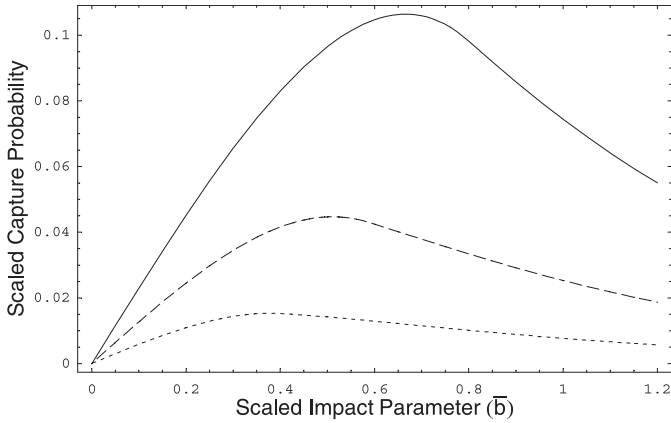
$$\begin{aligned} \bar{b} \bar{P}_C(\bar{b}, \bar{E}, \bar{\lambda}, a_A) &= \frac{8\bar{R}_C(\bar{E}, \bar{\lambda}, a_A) \bar{b}}{3\bar{E}^{1/2}} \\ &\quad \times \int_0^{\infty} dQ_{\perp} \frac{J_0(Q_{\perp} \bar{b}) J_1(Q_{\perp} \bar{R}_C(\bar{E}, \bar{\lambda}, a_A))}{Q_{\perp}^2 + 4}, \end{aligned} \quad (13)$$

where  $\bar{b}$  ( $\equiv b/a_Z$ ) is the scaled impact parameter and  $Q_{\perp}$  ( $\equiv q_{\perp} a_Z$ ) is the scaled perpendicular momentum transfer. Hence, the collective plasma screening and quantum effects on the electron capture process in strong coupled plasmas can be found from equation (13) with equations (4) and (5).

In order to explicitly investigate the quantum and collective screening effects on the electron capture probability, we set the collision energy as  $\bar{E} = 2$  since the Bohr-Lindhard method is known to be reliable for the region  $v_P > v_Z$  [2,6]. Figure 1 shows the scaled electron capture radius as a function of the scaled de Broglie wavelength for various values of the scaled reciprocal Debye length. Figure 2 represents the three-dimensional plot of scaled capture radius as a function of the scaled de Broglie wavelength and the scaled reciprocal Debye length. As we can see in these figures, the electron capture radius decreases with increasing the de Broglie wavelength, i.e., increasing the quantum effect. It is also found that the electron capture radius decreases with decreasing the reciprocal Debye length, i.e., increasing the plasma screening effect. However, the quantum effects are found to be more effective than the plasma screening effects. Figure 3 shows the scaled electron capture probability as a function of the scaled impact parameter for various values of the scaled de Broglie wavelength. In addition, Figure 4 shows the three-dimensional plot of the scaled electron

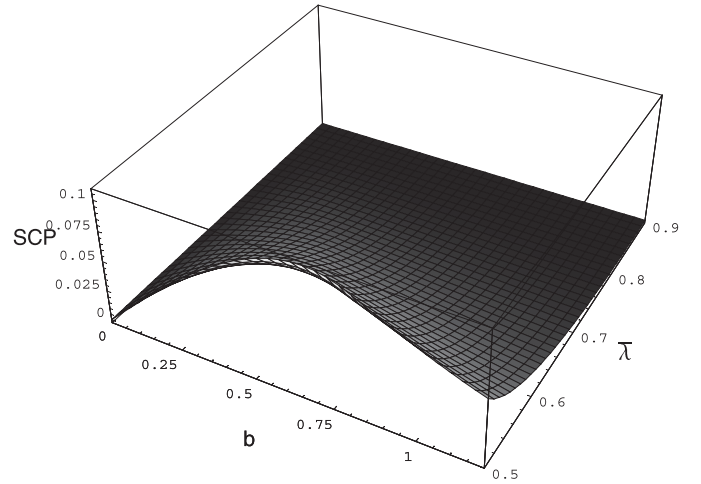


**Fig. 2.** The three-dimensional plot of scaled capture radius (SCR) ( $\bar{R}_C$ ) as a function of the scaled de Broglie wavelength ( $\bar{\lambda}$ ) and the scaled reciprocal Debye length ( $a_\Lambda$ ).

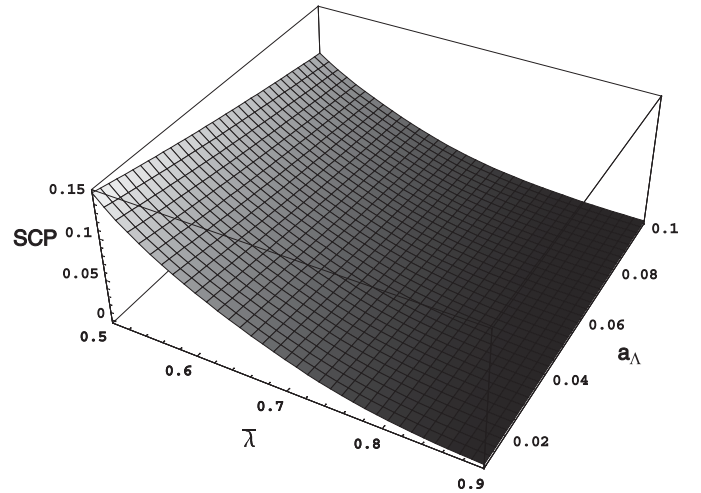


**Fig. 3.** The scaled electron capture probability ( $\bar{b}\bar{P}_C$ ) as a function of the scaled impact parameter ( $\bar{b}$ ) for various values of the scaled de Broglie wavelength ( $\bar{\lambda}$ ) when  $a_\Lambda = 0.1$ . The solid line represents the electron capture probability for  $\bar{\lambda} = 0.5$ . The dashed line represents the electron capture probability for  $\bar{\lambda} = 0.6$ . The dotted line represents the electron capture probability for  $\bar{\lambda} = 0.7$ .

capture probability as a function of the scaled impact parameter and the scaled de Broglie wavelength. From these figures, we can find that the maximum position of the electron capture probability, i.e., the position at which takes place the electron capture process, is shifted to the core of the projectile ion with increasing the de Broglie wavelength. Figure 5 represents the three-dimensional plot of the scaled electron capture probability as a function of the scaled de Broglie wavelength and the scaled reciprocal Debye length at  $\bar{b} = 0.5$ . From this figure, it can be also found that the quantum effects on the electron capture probability are more significant than the collective plasma screening effects on the electron capture probability in strongly coupled plasmas. The electron capture probability can be expressed as the following form in or-



**Fig. 4.** The three-dimensional plot of the scaled electron capture probability (SCP) ( $\bar{b}\bar{P}_C$ ) as a function of the scaled impact parameter ( $\bar{b}$ ) and the scaled de Broglie wavelength  $\bar{\lambda}$ .

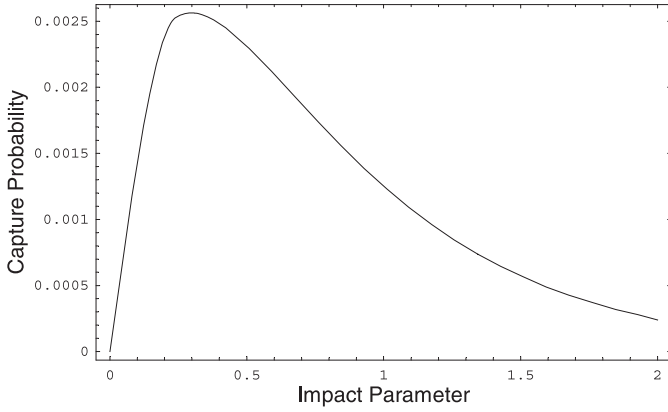


**Fig. 5.** The three-dimensional plot of the scaled electron capture probability (SCP) ( $\bar{b}\bar{P}_C$ ) as a function of the scaled de Broglie wavelength ( $\bar{\lambda}$ ) and the scaled reciprocal Debye length ( $a_\Lambda$ ) at  $\bar{b} = 0.5$ .

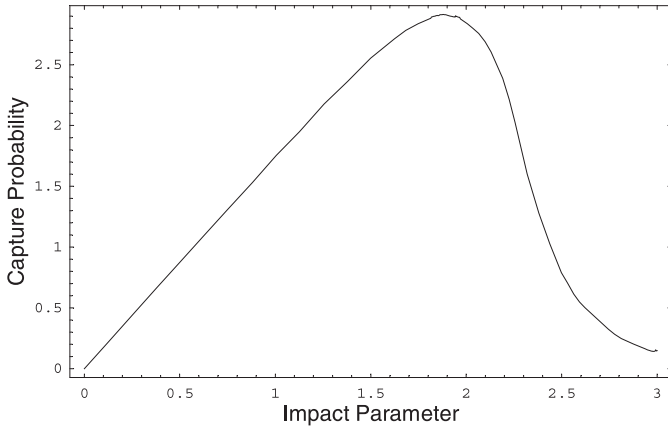
der to investigate the dependence of the electron capture probability on the projectile charge:

$$\bar{b}\tilde{P}_C(Z, \tilde{b}, \tilde{E}, \tilde{\lambda}, \tilde{a}_\Lambda) = \frac{8Z^3\tilde{R}_C\tilde{b}}{3\tilde{E}^{1/2}} \times \int_0^\infty d\tilde{Q}_\perp \frac{J_0(\tilde{Q}_\perp\tilde{b})J_1(\tilde{Q}_\perp\tilde{R}_C)}{\tilde{Q}_\perp^2 + 4Z^2}, \quad (14)$$

where  $\tilde{R}_C(Z, \tilde{E}, \tilde{\lambda}, \tilde{a}_\Lambda) = \ln\left(4 - \sqrt{21 - 12\tilde{F}}\right)^{-1/\tilde{B}}$ ,  $\tilde{E} \equiv E/Ry$ ,  $\tilde{\lambda} \equiv \lambda/a_0$ ,  $\tilde{a}_\Lambda \equiv a_0/\Lambda$ ,  $\tilde{Q}_\perp \equiv q_\perp a_0$ ,



**Fig. 6.** The electron capture probability ( $\tilde{b}\tilde{P}_C$ ) as a function of the impact parameter  $\tilde{b}$  ( $\equiv b/a_0$ ) for  $Z = 1$  when  $\tilde{\lambda} = 3$ ,  $\tilde{a}_A = 0.1$ , and  $\tilde{E} = 0.5$ .



**Fig. 7.** The electron capture probability ( $\tilde{b}\tilde{P}_C$ ) as a function of the impact parameter  $\tilde{b}$  ( $\equiv b/a_0$ ) for  $Z = 2$  when  $\tilde{\lambda} = 3$ ,  $\tilde{a}_A = 0.1$ , and  $\tilde{E} = 0.5$ .

$$\tilde{B}(\tilde{\lambda}, \tilde{a}_A) = \sqrt{[1 + \sqrt{1 - (2\tilde{\lambda}\tilde{a}_A)^2}]/(2\tilde{\lambda}^2)}, \text{ and}$$

$$\tilde{F}(Z, \tilde{E}, \tilde{\lambda}, \tilde{a}_A) = \frac{(\tilde{E}/Z\sqrt{2})\tilde{\lambda}\sqrt{1 - (2\tilde{\lambda}\tilde{a}_A)^2} + \sqrt{1 - \sqrt{1 - (2\tilde{\lambda}\tilde{a}_A)^2}}}{\sqrt{1 + \sqrt{1 - (2\tilde{\lambda}\tilde{a}_A)^2}}}. \quad (15)$$

Figures 6 and 7 show the variation of the electron capture probability with changing the charge state. As we see in these figures, the electron capture probability is significantly increased with an increase of the charge. Recently, the charge exchange process was investigated in weakly coupled classical plasmas [25] using the Bohr-Lindhard analysis [11] including only the plasma screening effects since the quantum effects are negligible in weakly coupled plasmas. However, in strongly coupled semiclassical plasmas the quantum effects play important roles as we see in Figures 1–7. Thus, equations (14) and (15) contain an important information on quantum effects as well as the

plasma screening effects on electron capture processes in strong coupled plasmas.

## 4 Summary and discussions

We investigate the quantum and collective screening effects on the electron capture processes by projectile ions from hydrogenic ions in strongly coupled semiclassical plasmas. The electron capture radius by the projectile ion in strongly coupled plasmas is obtained by the screened pseudopotential model taking into account both the plasma screening and quantum effects. The semiclassical version of the Bohr-Lindhard method is used to obtain the electron capture probability. The impact-parameter trajectory analysis is applied to the motion of the projectile ion in order to visualize the electron capture radius and the capture probability as functions of the impact parameter, thermal de Broglie wavelength, Debye length, and collision energy. It is found that the quantum and plasma screening effects significantly reduce the electron capture probability as well as the electron capture radius. It should be noted that the maximum position of the electron capture probability, i.e., the position at which takes place the electron capture process, is shifted to the core of the projectile ion with increasing the thermal de Broglie wavelength, i.e., increasing the quantum effect. It is also found that the quantum effects on the electron capture probability are more significant than the plasma screening effects on the electron capture probability in strongly coupled plasmas. The electron capture probability is found to be significantly increased with an increase of the charge. These results provide useful information on electron capture processes in strongly coupled semiclassical plasmas.

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